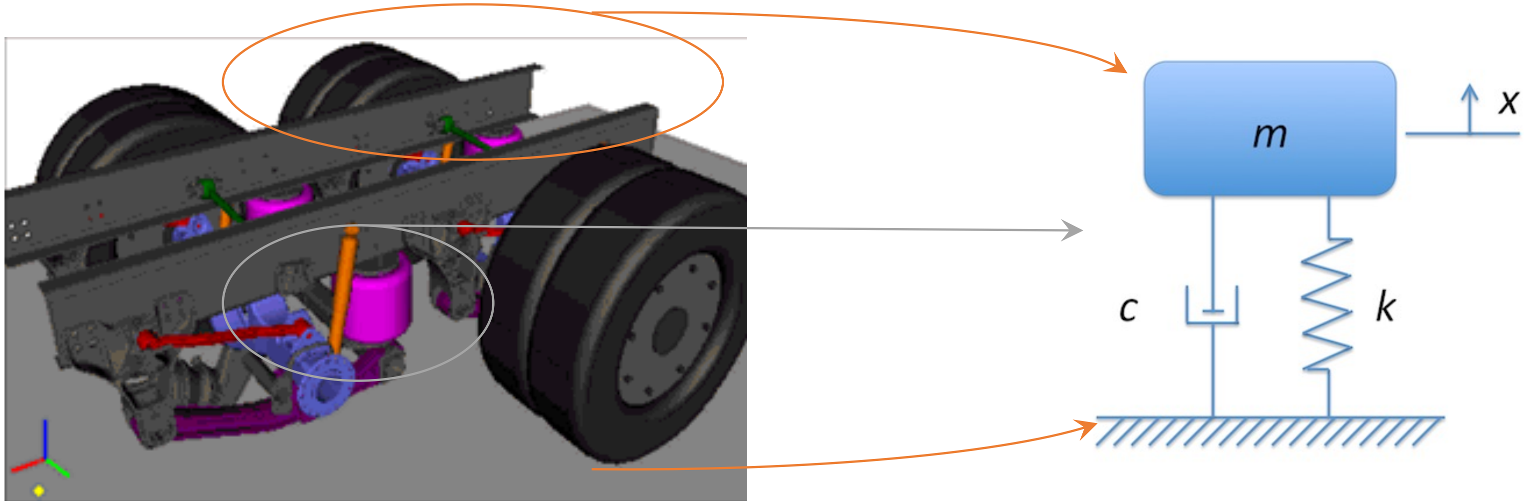
EPFL

(short) Intro to Lumped Element Modelling

micro-470

2024

Mechanical Example



- What did we neglect?
- What did we gain ?
- Genius or stupid approach?

Lumped Element Model (LEM) allows simplifying systems

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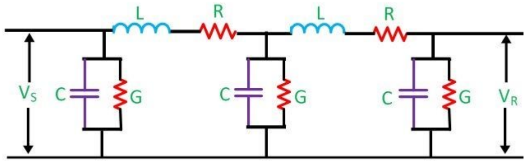
- A lumped element model (LEM) is a way to simplify a given system into a combination of discrete elements (“lumped”)
  - The elements can exchange energy/information with other elements
  - Signals/Variables propagate instantaneously inside the elements
- This approach greatly reduces the complexity of the system
- In some cases, the model will give a good approximation (but can also give nonsense if physics poorly understood)

Lumped element = information travels instantaneously through device



Lumped element = information travels instantaneously through device

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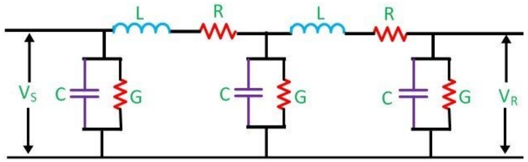
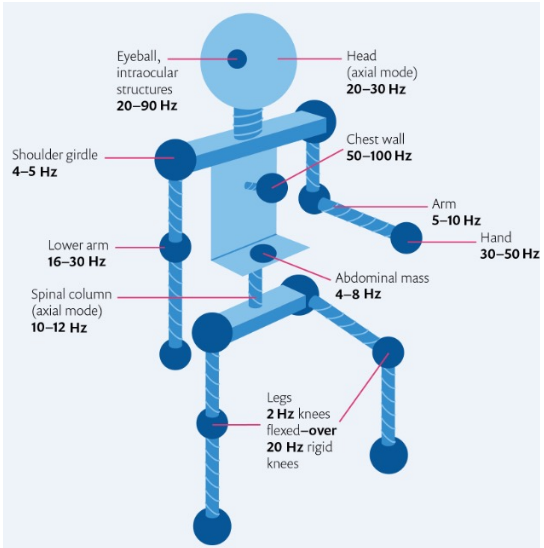
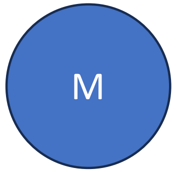


https://www.youtube.com/watch?v=7Ht5m2iwDys



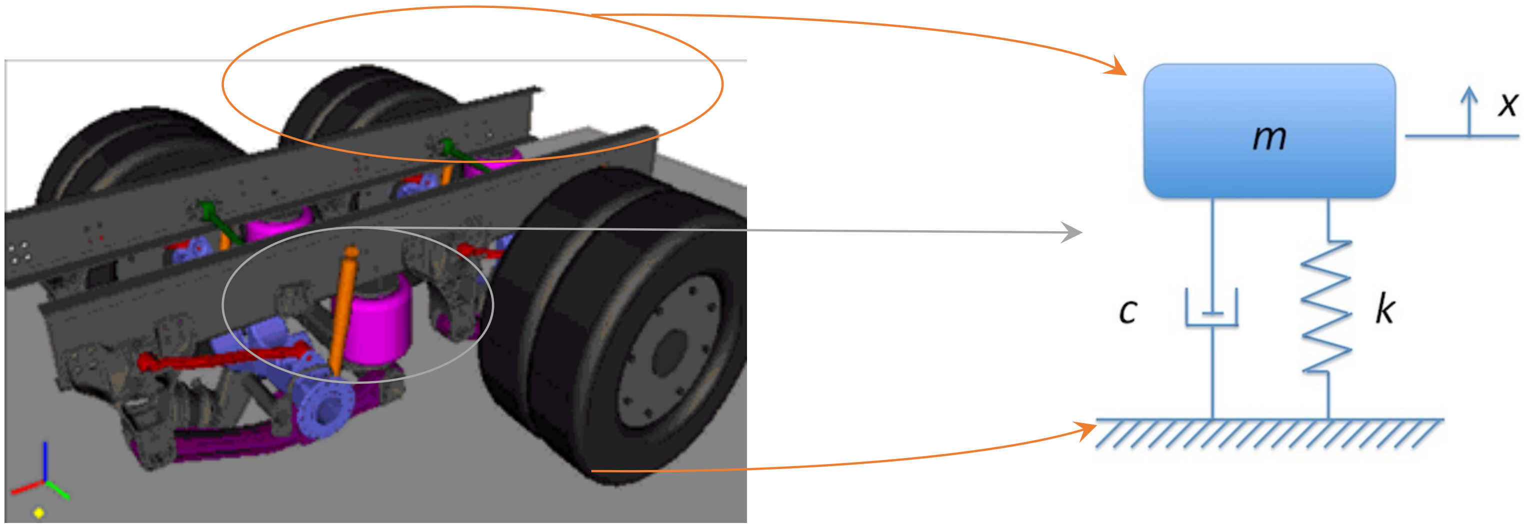
https://www.comsol.com/blogs/how-to-use-lumped-elements-to-model-a-mechanical-system/



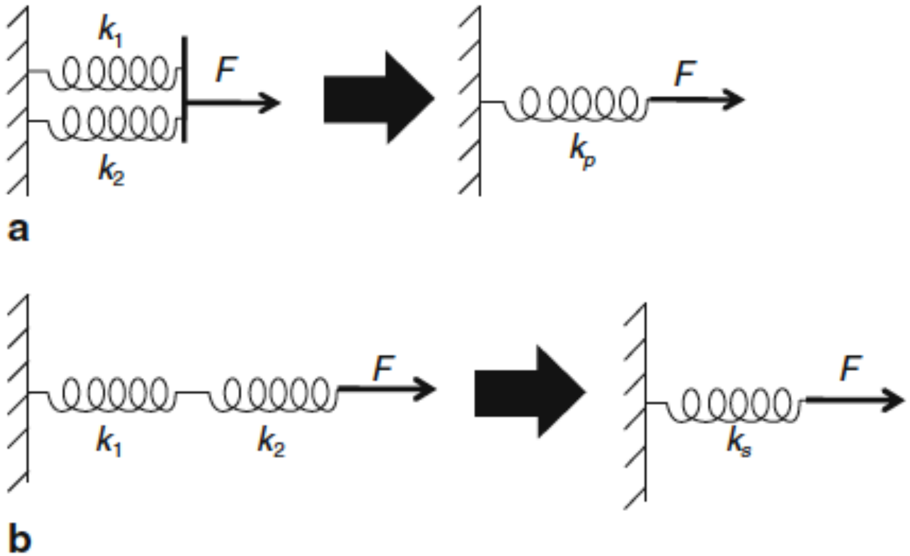
Mechanical Example

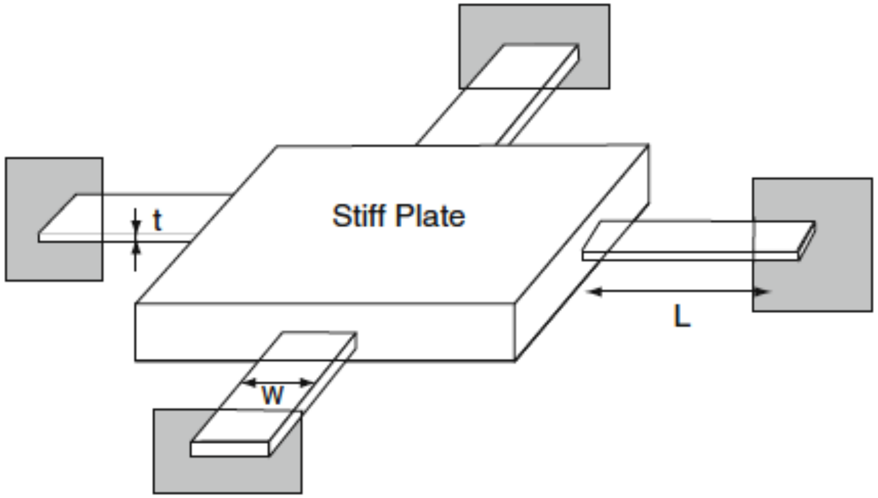
- Variables: position (x) and velocity ($\dot{x}$) of the mass
- Mass connected to ground via spring and damper
  - Approximations: Mass is rigid and concentrated; spring and damper are massless
- Eq of Motion: $m\ddot{x} + c\dot{x} + kx = F(t)$

Solve in Matlab, C++, or analytically
If make equivalent circuit, can use Circuit simulator



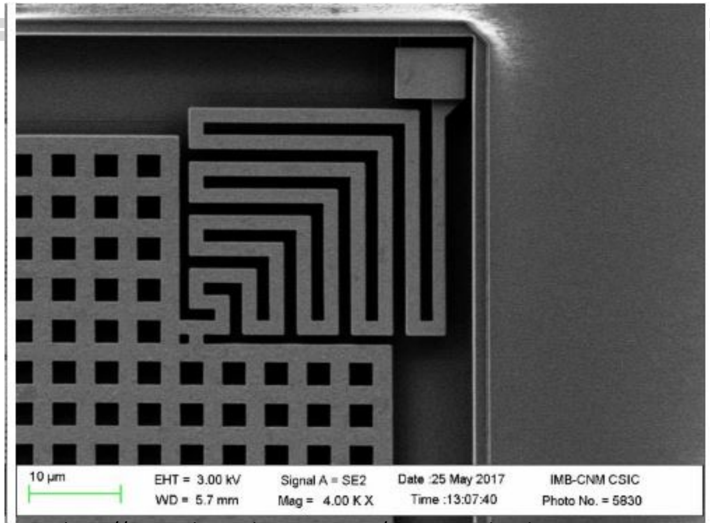
Complex springs can be modeled as combination of simple elements



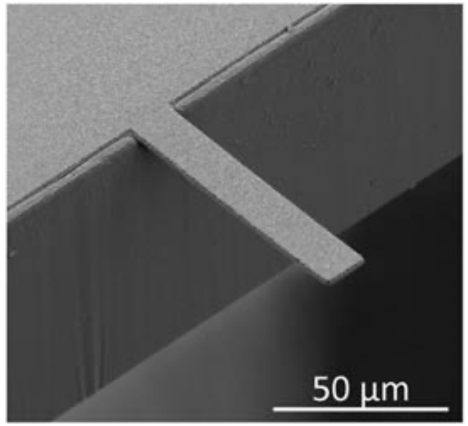


Springs are more complex than $F = kx$…

- Simple Cantilever has
  - Many bending modes, torsion, etc.
  - Anchor effects
  - Material non-linearities
  - Depends on how load is applied
  - **Yet in Lumped Element Modeling (LEM), we will represent a complex deformation as a simple spring!**
  - **LEM is powerful** as we includes key cantilever dimensions as parameters, allowing quick design of full system, with analytical or numerical solutions
  - **But a useful LEM requires some physical insight!**



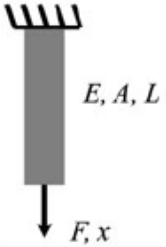
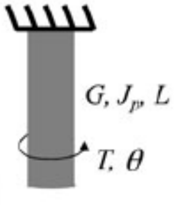
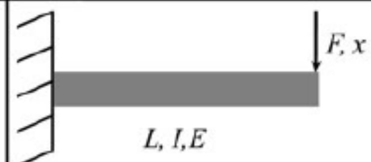
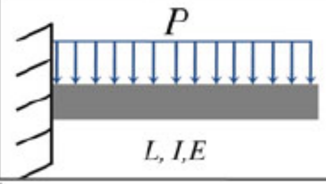
http://www.chipscalereview.com/news1708.html



http://microchem.com/Appl-MEMs-Cantilevers.htm

Several different *k* for a simple cantilever, depending on how it is deformed…

Table 4.1 The linear stiffness coefficient k of some of the common structure configurations in MEMS. The unit of k for all loading types is force/length. For wide beams ($b > 5h$) of cases 3–9, replace E with $E/(1 - \nu^2)$, where ν is the Poisson's ratio. When calculating the natural frequency of a flexible structure alone, use an "effective" mass for the structure

1) Axially loaded bar		$k = \frac{EA}{L}$ E: Modulus of elasticity A: Area of cross section L: Length of bar
2) Rod under torque		$k = \frac{GJ_p}{L}$ J_p: Polar moment of inertia of the cross section G: Shear modulus L: Length of rod
3) Cantilever beam under point load at the tip		$k = \frac{3EI}{L^3}$ E: Modulus of elasticity I: Moment of inertia of the cross section L: Length of bar
4) Cantilever beam under uniformly distributed pressure		$k = \frac{8EI}{L^3}$

For one spring, many spring constants, depending on:

- how beam bends, and
- where force is applied.

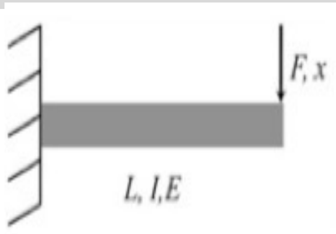
Younis, M. I. *MEMS Linear and Nonlinear Statics and Dynamics*. **20,** (Springer US, 2011)

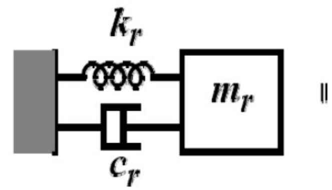
Lumped-parameter modeling in mechanics

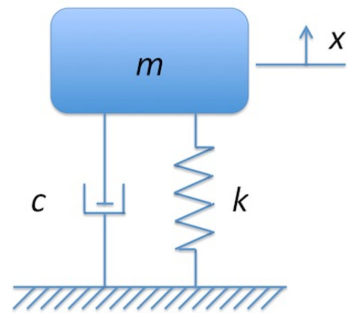
- the dynamics of mechanical structures can be reduced to a mass-spring model where the natural pulsation is given by

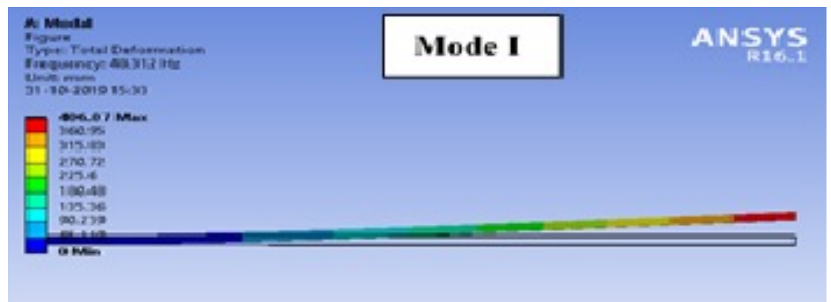
$$\omega_0 = \sqrt{k/m}$$

- k is given by calculation such as on previous slide
- But what mass to use? Seems obvious, but ….
- For each mode, we define an **effective mass,** m^*, that gives the appropriate natural frequency for a given structure of stiffness k.
- m^* is smaller than m for modes where entire mass is not moving









see: "Elements of Lumped-Parameter Modeling in MEMS", in M. I. Younis, MEMS Linear and Nonlinear Statics and Dynamics

Effective mass for a cantilever (point load at tip)

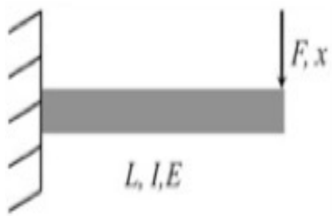
- exact analytical calculation of the natural freq. of a cantilever: $$f_0 = \frac{1}{2\pi}\frac{h}{l^2}\sqrt{\frac{E}{\rho}}$$

$$\omega_0 = \sqrt{k/m}$$

E: young's modulus
ρ: density
h: thickness
l: length
w: width

- natural frequency in equivalent effective mass-spring model: $$f_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m*}}$$

- By taking the stiffness of the cantilever for end loading: $$k = \frac{Ewh^3}{4l^3} \quad \propto L$$



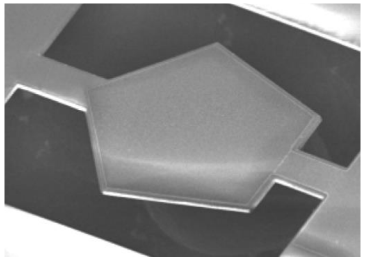
- Effective mass of 1st vibration mode of the cantilever : $$\boxed{m^* = \frac{1}{4}\rho \cdot w \cdot h \cdot l \quad = m/4}$$

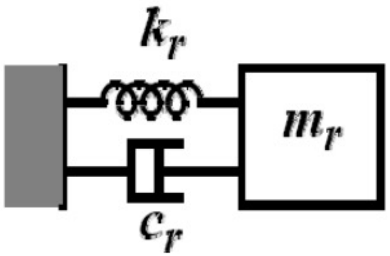
see: Elements of Lumped-Parameter Modeling in MEMS, in M. I. Younis, MEMS Linear and Nonlinear Statics and Dynamics

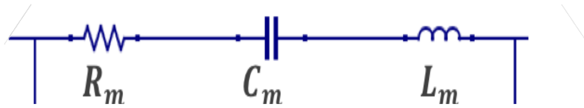
Electrical equivalent circuit

- Very often, we will work in the electrical domain, as very easy to solve electrical circuits (using p-spice, QUCS, etc)

Mechanical Variable	Electrical Variable
Damping, c	Resistance, R
Stiffness^{-1}, k^{-1}	Capacitance, C
Mass, m	Inductance, L
Force, f	Voltage, V
Velocity, v	Current, I



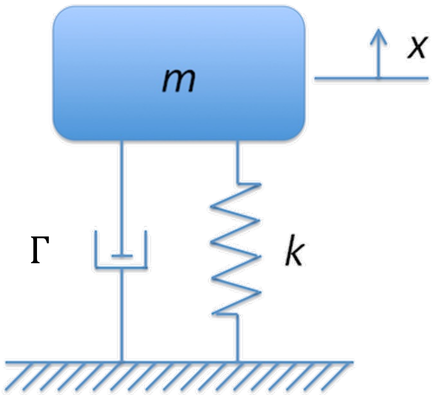




Equivalent circuit – Generalized variables

Generalized	Electrical case	Mechanical case	Fluidic case	Thermal case
Displacement – q	Charge – q	Displacement – x	Volume – V	Heat – Q
Flow – f	Current – I	Velocity – v	Flow rate – Q	Heat flow – $\dot{Q}$
Effort – e	Voltage – V	Force – F	Pressure – P	T difference – ΔT
Momentum – p	–	Momentum – p	Momentum – Γ	–
Resistance – R	Resistor – R	Damper – c	Fluid resist. – R	Thermal resist. – R
Capacitance – C	Capacitor – C	Spring – $1/k$	Fluid capac. – C	Thermal capac. – C
Inertance – L	Inductor – L	Mass – m	Inertance – m	–
Node law	Kirchoff's	Continuity of space	Mass conservation	Heat conservation
Mesh law	Kirchoff's	Newton's 2nd	Relative P	Relative T

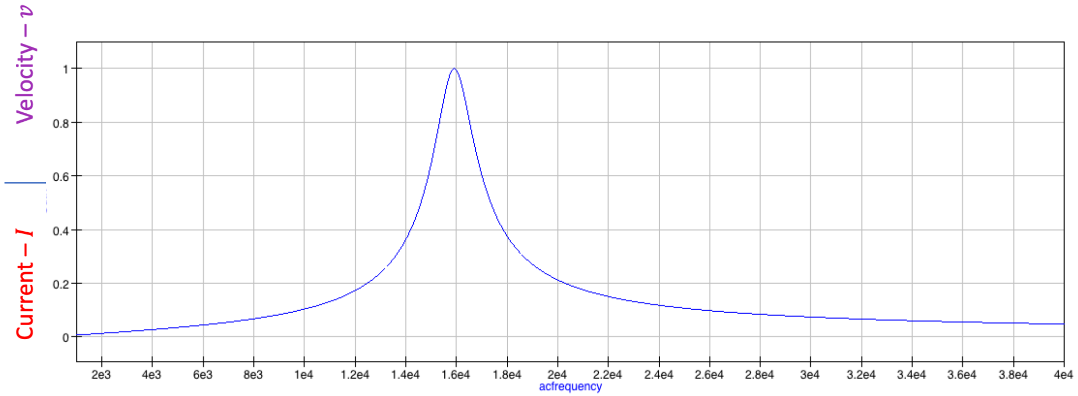
Equivalent circuit for mechanical resonator



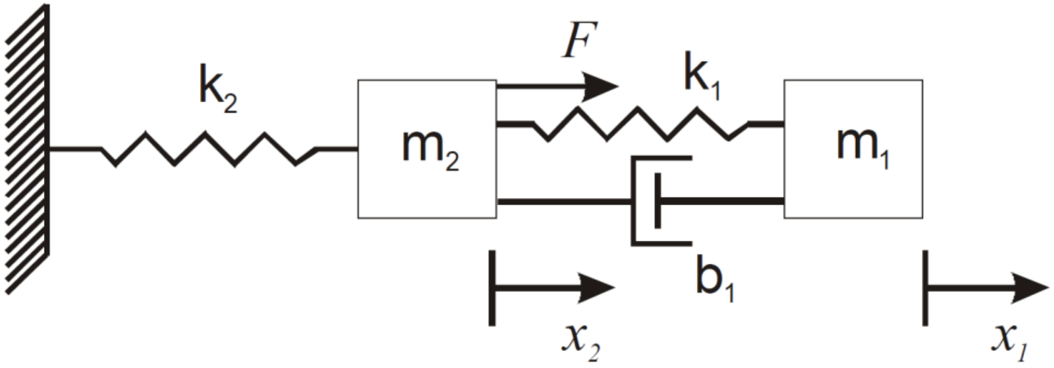
$$L \leftrightarrow m \qquad R \leftrightarrow \Gamma \qquad C^{-1} \leftrightarrow k$$

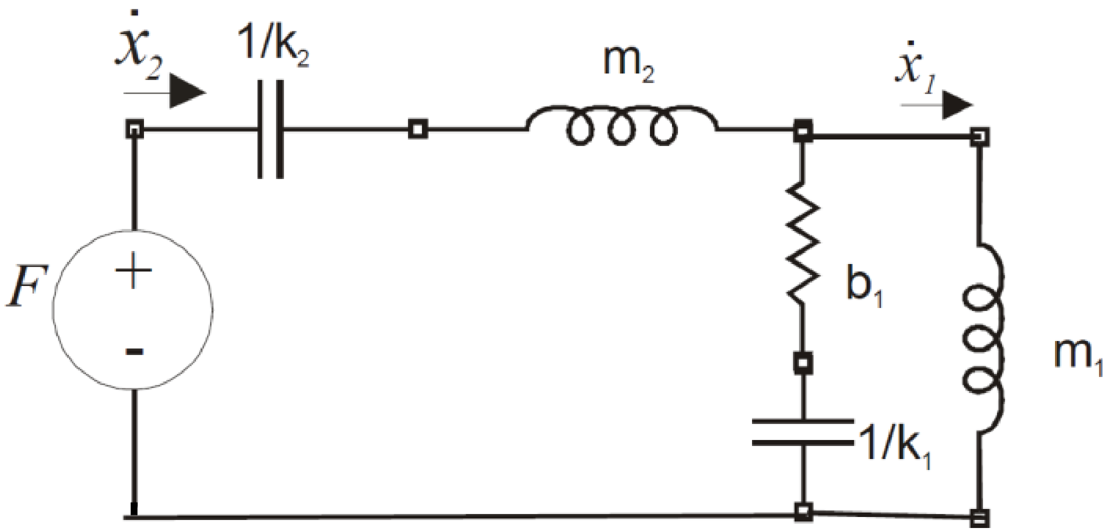
$$\omega_0 = \sqrt{\frac{k}{m}} \qquad Q = \frac{\sqrt{km}}{\Gamma}$$

$$\omega_0 = \sqrt{\frac{1}{LC}} \qquad Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$



Example of mechanical LEM to electrical LEM

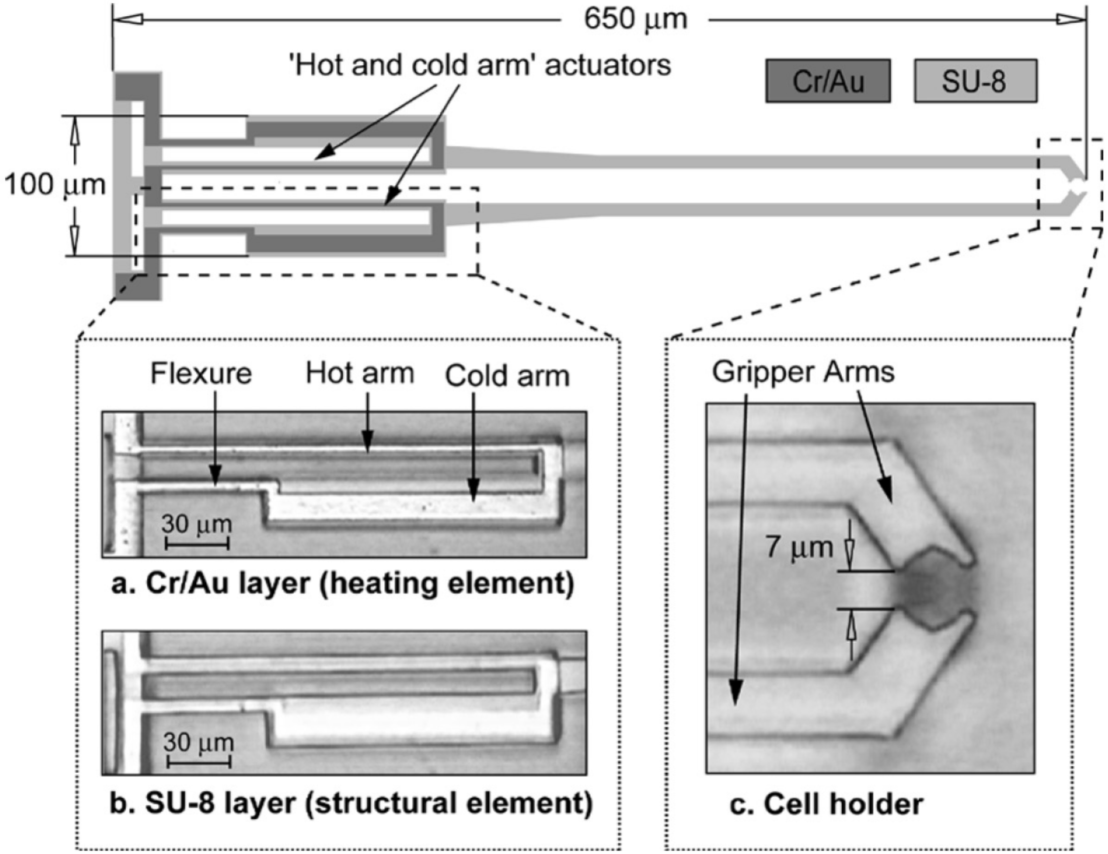




Can do LEM for Thermal, Fluidic, etc

- Usefulness of LEM is
  - Very fast simple model
  - Couple physics and solve all in circuit simulator

Example of Coupled thermal-mechanical LEM



1. Thermal model of hot arm (find T_{arm} vs. V_{drive})
2. Mechanical model of actuator (find F vs. T_{arm})
3. Link them to obtain *F* vs. V_{drive}

The Output temperature (a voltage in LEM) serves as input drive force (also a voltage in LEM)

N. Chronis and L. P. Lee, "Electrothermally activated SU-8 microgripper for single cell manipulation in solution," Journal of Microelectromechanical Systems, vol. 14, pp. 857–863, 2005

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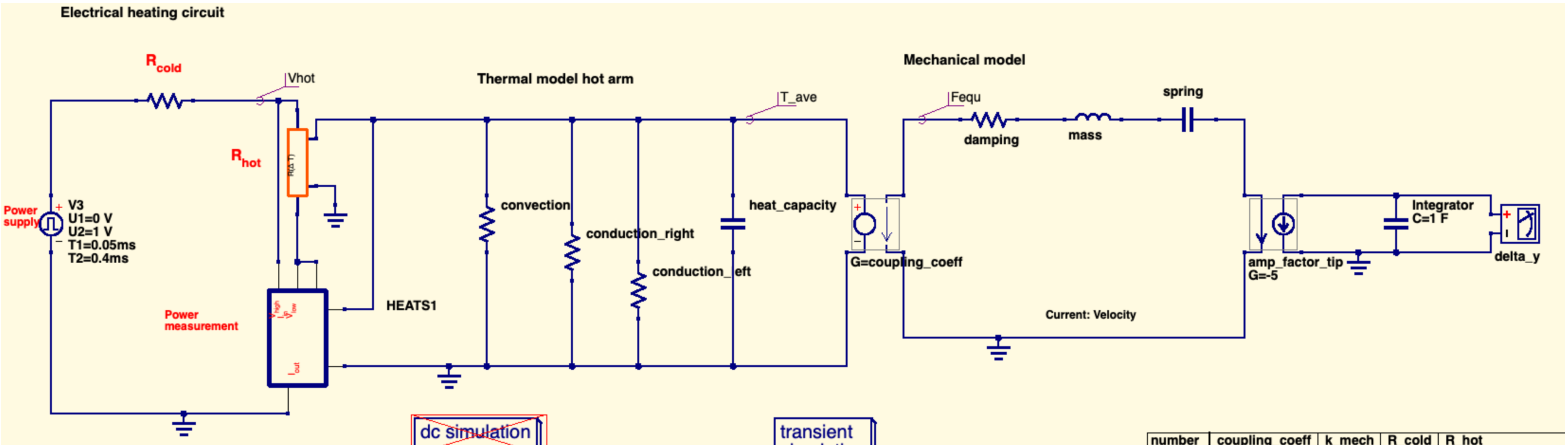
Example thermal + mechanical LEMs - > circuit



- Output is: dynamics of gripper closing vs input voltage to heater
- Can explore parameter space of spring, thermal mass, etc, really fast

LEM was made by A. Vaillancourt and S. Stettler in micro-402 class in 2017-2018.

- LEM Useful for quickly exploring parameter space and different configurations
- LEM good to combine physics
- Then for detailed dimensioning, use FEM